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Group Theory of Spontaneous Symmetry Breaking

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The connection between the minimality of the Higgs field potential and the maximal little groups of its representation obtained by spontaneous symmetry breaking is analyzed. It is shown that for several representations the lowest minimum of the potential is related to the maximal little group of those representations. Furthermore, a practical necessity criterion is given for the representation of the Higgs field needed for spontaneous symmetry breaking.

1. INTRODUCTION

Since the success of GSW theory (Glashow, 1961; Glashow *et al.*, 1970; Salam, Weinberg, 1967) based on spontaneous symmetry breaking (SSB) [Higgs mechanism (Higgs, 1964a,b)] and related GUTs with mass production through SSB, there has been a great deal of effort to understand the group-theoretic aspects of SSB and its possibilities for unifying the physical interactions by incorporating supersymmetry.

The pioneering work in this direction of Michel (1979) and Li (1974) was followed by others generalizing and explaining various aspects of SSB (Ruegg, 1980; Bucella *et al.*, 1980; Kim, 1982). We give here a brief review of the SSB mechanism for a group (algebra) G. Generally for a G-invariant Lagrangian, SSB is possible when a Higgs field φ belonging to a representation[2] r_G of G acquires a vacuum expectation value (vev) $\langle\varphi_{r_G}\rangle$ invariant only with respect to a subgroup $G_i \subset G$. Three *dependent* conditions must be fulfilled in a SSB $G \to G_1$:

I. Invariance of r_G with respect to G_1, $G_1(r_G) = r_G$

II. Minimization of the potential by $\langle\varphi r_G\rangle$.

III. Mass generation for the gauge bosons related to (G/G_1) by φ_{r_G}.

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[2]Here we discuss only the single irreducible representation (irrep.) of unitary groups; for the cases of the combined irreps see footnote 7. The method can be used immediately for the orthogonal groups.

Conditions I and III can be fulfilled if G_1 is a little group of r_G, but II is generally (!) fulfilled only if G_1 is a maximal little group (m.l.g.)[3] of r_G. The quantitative structure of φ_{r_G} arising from the minimization procedure determines the directions of SSB (the rank distribution of unbroken groups or little groups).

Given the kind[4] of representation r_G for the Higgs field, without any need for minimization of the potential or detailed knowledge of the $\langle\varphi_{r_G}\rangle$, it is obvious that the remaining gauge group is an element of the set of little groups of r_G. But because of free parameters in the potential $V(\varphi_{r_G})$ one cannot immediately determine the physical unbroken gauge group. So it is of interest to find a general criterion for determining a certain subset of little groups as the possible domain of remaining gauge groups.

2. SINGLET CRITERION

A singlet criterion[5] based on condition I requires that the decomposition of r_G with respect to G_1 must have only one singlet, because the invariance I requires at least one singlet of r_G with respect to G_1 "as a necessary condition for SSB."[6] More than one singlet of r_G with respect to G_1 in a

[3]In the case of $G_1 = SU(n)\times SU(m)$ we define the maximal little groups so that the decomposition of the $SU(N)$ representation with respect to them has the smallest number of singlets (see footnote 8), i.e., generally one; or equivalently those subgroups with $n_1+n_2=N$, where n_1 and n_2 have to be determined for each kind of representation differently. The criterion that "maximal little groups must not contain each other" holds only for the adjoint representations, but obeys our criterion, too. In the last time there are some counterexamples to the "Michel conjecture," but we do not discuss this here.

[4]Vector, adjoint, or tensors, without quantitative vacuum structure.

[5]Introduced in Ghaboussi (1982). For example, to break $SU(N)\to SU(2)\times SU(N-2)$, one needs an antisymmetric second-rank tensor, because (in dimension)

$$SU(N)\subset SU(2)\times SU(N-2)$$
$$\binom{N}{2} = (1,1)+(2,N-2)+\cdots$$

[6]Because then one can choose the whole of the nonsinglet parts of the decomposition (see footnote 7) as equal to zero, to have enough freedom to demonstrate the desired invariance within the nonzero part of the singlet. For example, in the case of the adjoint representation of $SU(5)$ one has

$$SU(3)\left[\begin{array}{ccccc} v & 0 & 0 & 0 & 0\\ 0 & v & 0 & 0 & 0\\ 0 & 0 & v & 0 & 0\\ 0 & 0 & 0 & -\frac{3}{2}V & 0\\ 0 & 0 & 0 & 0 & -\frac{3}{2}V \end{array}\right] SU(2)$$

SSB (decomposition) is equivalent to introduction of more than one irrep to produce the same SSB; but in this case the representation is no longer single.[7] On the other hand, the requirement of only one singlet is equivalent to demanding that G_1 must be a maximal little group of r_G.[8] So our singlet criterion implies the Michel (1979) conjecture.

The domain of possible unbroken gauge groups in $G \to G_1$ is bounded to the set of maximal little groups of r_G; but the determination of the physical G_1 from this set of groups depends generally on the structure of the potential and in particular on the sign of the leading order of the potential polynomial.[9]

It is the aim of this paper to clarify the relation between the minimalization of potential by a numerical representation $\langle r_n \rangle$ and the maximality of its little groups: $\langle r_n \rangle \in \{\langle r_G \rangle\}$,

$$[V(\langle r_n \rangle)\colon V_{\text{minimum}}] \to [G_1(\langle r_n \rangle) = \langle r_n \rangle,\ G_1 \in \{m.l.g.\}_{\langle r_G \rangle}]$$

We discuss now the usual case of SSB in gauge theories, $N = n_1 + n_2$, $SU(N) \to SU(n_1) \times SU(n_2)$.

[7]For example,

$$SU(5) \overset{?}{\to} SU(3) \times SU(2) \times U(1)$$
$$(24)_1 = (1,1) + (8,1) + (\bar{3},2) + (1,3) + (3,2)$$
$$SU(5) \overset{?}{\to} SU(2) \times SU(2) \times U(1) \times U(1)$$
$$(24)_2 = (1,1) + (1,1) + (3,1) + (2,1) + (2,1) + \cdots$$

and this means

$$SU(5) \xrightarrow{(24)_1} SU(3) \times SU(2) \times U(1) \xrightarrow{(24)_2} SU(2) \times SU(2) \times U(1) \times U(1)$$

Generally the cases with more than one irrep, $r_G = r_1 + r_2 + \cdots$, are equivalent to the case obtained by introducing the r_1 and r_2 step by step, i.e.,

$$G \xrightarrow{r_1} G_1 \xrightarrow{r_2} G_2 \xrightarrow{\cdots} \cdots$$

See also Slansky (1981).

[8]Only in the one case of antisymmetric representation may one have more than one singlet with respect to the maximal little groups. For example,

$$SU(N) \supset SU(n) \times SU(m), \qquad n = m = N/2$$
$$\binom{N}{n} = (1,1) + \cdots + (1,1),$$

[9]Here we are dealing with potentials with "relatively" fixed parameters, but we discuss the general case of variable parameters later.

3. QUALITATIVE ANALYSIS

We know that the general renormalizable $SU(N)$-invariant potential, first without a cubic term, with representation (r),[10] is

$$V(\varphi_r) = -\alpha \operatorname{Tr}(\varphi_r^2) + \beta[\operatorname{Tr}(\varphi_r^2)]^2 \pm \gamma \operatorname{Tr}(\varphi_r^4); \qquad \alpha, \beta, \gamma \rangle 0 \tag{1}$$

It is then plausible to state that the magnitude of minima of $V(\varphi_r)$ depends on the extrema of $\operatorname{Tr}(\varphi_r^4)$. In other words, the stable minimum of $V(\varphi_r)$ for $(+\gamma)$ occurs when $\operatorname{Tr}(\varphi_r^4)$ has its minimum, and for $(-\gamma)$ it occurs with maximum of $\operatorname{Tr}(\varphi_r^4)$.

Now the reason why the minimum of $V(\varphi_r)$ happens in general if the little group of $(r\colon r_n)$ is maximal is that the mentioned extremum of $\operatorname{Tr}(\varphi_r^4)$ (related to $V_{\min}$) occurs, as we show later, only for the extrema of

$$n = n_1/n_2 \tag{2}$$

for $(r\colon r_n)$ if

$$n_1 + n_2 = N \tag{3}$$

where n_1 and n_2 are the numbers of repeated eigenvalues in $\langle\varphi_{r_n}\rangle$. But (3) is the condition of maximal little groups of $SU(N)$-irreps,[11]

$$SU(n_1)\times SU(n_2) \subset SU(N) \tag{4}$$

because an $n_1 + n_2 = N$ structure of $\langle\varphi_{r_n}\rangle \in \{\langle\varphi_{r_G}\rangle\}$ demonstrates its invariance with respect to $SU(n_1)\times SU(n_2)$ as its maximal little group (m.l.g.). Usually for a group G (not necessarily unitary), a G-invariant polynomial like (1) can be considered as a function of (n_i) partitions of its dimension N,

$$V(\varphi_r) \propto \sum_l f_l(n_i) \tag{5}$$

and the extremalization of (5) requires those partitions so that their representations (φ_{r_n}) may acquire maximal little groups (see also Michel, 1979). Moreover, the extremalization of (5) requires then also the maximalization of little groups, i.e., maximal little groups $n_1 + n_2 = N$. We show this explicitly for the mentioned cases (see footnote 10). We note that the aim of this paper is to show that there exist well-defined potentials suitable to producing derived channels of SSB, i.e., with m.l.g. as the rest symmetry. But of course one can construct a potential that gives rise to other SSB channels.

[10] We discuss the adjoint and tensor representations. The generalization to the case of vector representation is straightforward.

[11] In the case of the adjoint representation the left side of (4) has an additional $U(1)$ factor, but without any influence on the proposed procedure.

4. THE ADJOINT REPRESENTATION OF $SU(N)$

As mentioned, the minimum of potential (1) depends on the extrema of $\mathrm{Tr}(\varphi_r^4)$. First we show that this extrema happens for just two different eigenvalues of the adjoint representation.

Lemma. In the case of adjoint representation, the extremum of $\mathrm{Tr}(\varphi_r^4)$ occurs for the smallest possible number of eigenvalues (obeys the tracelessness condition): two.

Equivalently the extrema of $\mathrm{Tr}(\varphi_{r_n}^4)$ are related to the m.l.g. of (φ_{r_n}).

Proof. Let us have three different eigenvalues; then

$$\mathrm{Tr}(\varphi_r^4) = \sum_{i=1}^{3} n_i a_i^4, \qquad \sum_{i=1}^{3} n_i = N, \qquad \sum_{i=1}^{3} n_i a_i = 0 \tag{6}$$

From (6) we have

$$\mathrm{Tr}(\varphi_r^4) = \left(\sum_{j=1}^{2} n_j a_j^4\right) + \frac{\left(\sum_{j=1}^{2} n_j a_j\right)^4}{(n_3)^3} \tag{7}$$

Now the extremum of (7) occurs for either

$$n_3 = 0, \qquad \sum_{j=1}^{2} n_j a_j = 0 \qquad \text{(maximum)} \tag{8}$$

or

$$n_3 = N-1, \qquad n_2 = 1, \qquad n_1 = 0 \qquad \text{(minimum)} \tag{9}$$

in other words, for $N = n_1 + n_2$.

It is proved that the interesting extrema of $\mathrm{Tr}(\varphi_r^4)$ occur if only two different eigenvalues exist:

$$n_1 + n_2 = N \tag{10}$$

and this means that the little groups of (φ_r) as mentioned above are maximal little groups (see footnote 11):

$$SU(n_1) \times SU(n_2) \quad (\times U(1)) \quad \blacksquare$$

Thus, it is shown that the minimum of the potential for the adjoint representation happens for $n_1 + n_2 = N$ and thereby for eigenvalues with maximal little groups [see singlet criterion and Michel's (1979) conjecture].[12]

[12]The adjoint representation of $SU(N)$ has just one singlet only with respect to $SU(n_1) \times SU(n_2) \times U(1)$ maximal little groups, i.e., $(n_1 + n_2 = N)$

$$SU(N) \supset SU(n_1) \times SU(n_2) \times U(1)$$

(in dimension)

$$(N^2 - 1) = (1, 1) + \sum_{\substack{a=1 \\ a \neq b = 1}} (a, b)$$

One can show much more, that the extrema of such a $\mathrm{Tr}(\varphi_r^4)$, and thereby the related potential minima, coincide directly with the extrema of the maximal little groups.[13]

For a fixed eigenvalue of $(\langle\varphi_{r_n}\rangle;\ n_1 \geq n_2, |a_1| = 1)$ we have

$$\mathrm{Tr}(\varphi_{r_n}^4) = n_1 a_1^4 + n_2 a_2^4, \qquad n_1 a_1 + n_2 a_2 = 0 \tag{11}$$

and then

$$\mathrm{Tr}(\varphi_{r_n}^4) = n_1 a_1^4 [1 + (n_1/n_2)^3] \tag{12}$$

The extremum of (12) occurs for the extremum of (n_1/n_2). So the extrema of $\mathrm{Tr}(\varphi_{r_n}^4)$ are

$$\mathrm{Tr}(\varphi_{r_n}^4)\begin{cases}\text{maximum}, & n_1/n_2 = N-1 \leftrightarrow (n_2 = 1)\\ \text{minimum}, & n_1/n_2 \leq N - [N/2]/[N/2] \leftrightarrow (n_2 = [N/2])\end{cases} \tag{13}$$

On the other hand, as mentioned before, the minimum of potential (1) depends on the minimum of $\mathrm{Tr}(\varphi_r^4)$ for $(+\gamma)$ and on the maximum of $\mathrm{Tr}(\varphi_r^4)$ for the $(-\gamma)$. Thus, the stable minimum of the potential occurs $(n_1 + n_2 = N)$:

$$V_{\min}(\langle\varphi_{r_n}\rangle) = \begin{cases}+\gamma \sim n_2 = [N/2]\\ -\gamma \sim n_2 = 1\end{cases} \tag{14}$$

This means that the exact directions of SSB with adjoint representation for potential (1) are

$$\begin{aligned} &SU(N) \to SU([N/2]) \times SU(N - [N/2]) \times U(1), \qquad +\gamma\\ &SU(N) \to SU(N-1) \times U(1), \qquad -\gamma \end{aligned} \tag{15}$$

5. GENERAL CASES

Now, if we choose α, β, and γ as variable parameters [for example, if β and γ increase with respect to α, or if we introduce a cubic term with a variable parameter $\lambda > 0$, i.e., $V := [(1) \pm \mathrm{Tr}(\varphi_r^3) \cdot \lambda]$, then it is obvious that the minimum of such a potential can vary between the discussed extrema of (15). Note that for the case $(-\gamma)$ the minimum of the new potential depends as mentioned on the maximum of $\mathrm{Tr}(\varphi_r^4)$, so the variations of other parameters with respect to γ have no influence on this minimum, because the mentioned variations can be relatively limited by minimality conditions.[14] But in the case of $(+\gamma)$ the minimum of the potential cannot be reached by the minimum of $\mathrm{Tr}(\varphi_r^4)$ (as in the case of $\lambda = 0$), because the most minimal potential could arise here if the positive terms, such as $+\gamma\,\mathrm{Tr}(\varphi_r^4)$ [or $+\lambda\,\mathrm{Tr}(\varphi_r^3)$] decrease and the negative terms, such as

[13]The extrema of maximal little groups means the extrema of the number of group generators.
[14]I.e., $\partial V(\varphi_i)/\partial\varphi_i = 0$ and $\partial^2 V(\varphi_i)/\partial\varphi_i\partial\varphi_j > 0$ and (γ) can be chosen relatively large.

Table I. Variation of the Minima of the Potential with $(+\gamma)$ with Respect to the Value of n_2

$+\lambda$	$-\lambda$
$[\mathrm{Tr}(\varphi^3_{r_n}), \mathrm{Tr}(\varphi^4_{r_n})]_{\min} \sim ((n)_{\min};\ n_2 = [N/2])$	$[\mathrm{Tr}(\varphi^4_{r_n})]_{\min} \sim ((n)_{\min};\ n_2 = [N/2])$
$\vdots$	$\vdots$
$[\mathrm{Tr}(\varphi^2_{r_n})]_{\max} \sim ((n)_{\max};\ n_2 = 1)$	$[\mathrm{Tr}(\varphi^3_{r_n}), \mathrm{Tr}(\varphi^2_{r_n})]_{\max} \sim ((n)_{\max};\ n_2 = 1)$

$V(\varphi) = (1) \pm \lambda\ \mathrm{Tr}(\varphi^3_r);\ n = n_1/n_2.$

$-\alpha\ \mathrm{Tr}(\varphi^2_r)$ [or $-\lambda\ \mathrm{Tr}(\varphi^3_r)$] increase. On the other hand, all of these terms alter as functions of (n_1/n_2). So the minimum of the potential indicates for some terms increasing and for others decreasing (n_1/n_2), but because of variable parameters these variations can be managed through the relative choice of $\lambda/\alpha\gamma$.

Table I gives the variation of the minima of the potential with $(+\gamma)$ with respect to the value of n_2. So the SSB directions change for increasing $\pm\lambda$ relative to α and γ, from

$$SU(N) \to SU([N/2]) \times SU(N-[N/2]) \times U(1) \tag{16a}$$

to

$$SU(N) \to SU(N-1) \times U(1) \tag{16b}$$

6. THE TENSOR REPRESENTATION OF $SU(N)$

We use (1) and the standard form of matrix representation for both symmetric or antisymmetric tensors,[15] because these are transformable to the general forms of tensor representations. Introducing an $N \times N$ symmetric (s) or antisymmetric (as) standard matrix with parameter (C) as the tensor representation for (r), then one has (see Appendix)[16]

$$\mathrm{Tr}(\varphi^4_r) = n_1 C^4\colon \quad \begin{cases} 1 \le n_{(\mathrm{s})} \le N, & n_1\colon n_{(\mathrm{s})} \\ 2 \le n_{(\mathrm{as})} \le [N/2], & n_1\colon n_{(\mathrm{as})} \end{cases} \tag{17}$$

But $C^2 \propto 1/n$, and so we have

$$\mathrm{Tr}(\varphi^4_r) \propto 1/n_1 \tag{18}$$

[15] We discuss the usual case of second-rank tensors.

[16] See the Appendix. In the case of tensors the n_2 in (n_1/n_2) refers to the number of elements in diagonal or block-diagonal parts with $(C = 0)$, but we are finally interested only in the $C \neq 0$ part and thereby in the variation of (n_1).

Now, as mentioned in Section 1, the minimum of potential the (1) requires extrema of $\mathrm{Tr}(\varphi_r^4)$, so we have

$$\begin{aligned}
&\{[V(\varphi_r)]_{\min} \sim [\mathrm{Tr}(\varphi_r^4)]_{\min}, +\gamma\} \\
&\quad \sim (n_1)_{\max}: \quad [V(\varphi_r)]_{\min} \sim \begin{cases} n_{(s)} = N, & n_2 = 0 \\ n_{(as)} = [N/2], & n_2 = 0, 1 \end{cases} \\
&\{[V(\varphi_r)]_{\min} \sim [\mathrm{Tr}(\varphi_r^4)]_{\max}, -\gamma\} \\
&\quad \sim (n_1)_{\min}: \quad [V(\varphi_r)]_{\min} \sim \begin{cases} n_{(s)} = 1, \; n_2 = N-1 \\ n_{(as)} = 2, \; n_2 = N-2 \end{cases}
\end{aligned} \tag{19}$$

Thus the SSB directions in these cases are

$$\begin{aligned}
&\langle r_{(s)} \rangle \begin{cases} SU(N) \to O(N), & +\gamma \\ SU(N) \to SU(N-1), & -\gamma \end{cases} \\
&\langle r_{(as)} \rangle \begin{cases} SU(N) \to [\otimes SU(2)]^{[N/2]} \subset \mathrm{Sp}(2[N/2]), & +\gamma \\ SU(N) \to SU(2) \times SU(N-2), & -\gamma \end{cases}
\end{aligned} \tag{20}$$

It is shown that the minimum of the potential requires extrema of $\mathrm{Tr}(\varphi_r^4)$, i.e., $N = n_1 + n_2$, and thereby they are related to the maximal little groups of the introduced representation (r), and this has been done without performing the usual minimalization procedure.

7. REMARKS

The method used above is general (it requires no explicit minimalization) and coincides with the results of the usual minimalization method. On the other hand, the minimalization method without a criterion for invariance of the minimizing representation with respect to the unbroken subgroups[17] can result in unproved statements.[18] The interesting feature of this method, namely investigating the leading term, is that it clearly demonstrates the relation between the minima of the potential polynomial and the maxima of little groups. Furthermore, it has the advantage that it can be generalized to other kinds of groups.

Finally, we comment on the remark of Slansky (1981) about the little groups that occur as unbroken groups in SSB, such as $SU(3) \to U(1) \times U(1)$ with an adjoint representation if we use the partial potential

$$V(\varphi_r) = -\alpha\, \mathrm{Tr}(\varphi_r^2) + \beta[\mathrm{Tr}(\varphi_r^2)]^2$$

[17]For example, singlet criterion or invariance relations.

[18]See Li (1971) and Billoire and Morel (1981) for the case of antisymmetric tensor representation (see also Ghaboussi, 1982).

Here the situation is rather different from the general case that we discussed, because the magnitude of the potential depends, apart from parameters, only on $\mathrm{Tr}(\varphi_r^2)$. Thus, if we look at the eigenvalue conditions (7)-(9), then because the minimum of the potential depends on one hand on the minimum of $+\beta[\mathrm{Tr}(\varphi_r^2)]^2$ or $\mathrm{Tr}(\varphi_r)^2$, then with respect to (7)-(9), we must have $n_3=2$. On the other hand, it depends on the maximum of $-\alpha\,\mathrm{Tr}(\varphi_r^2)$, requiring $n_3=0$. Thus, for a suitable α/β it is plausible that one can have a compromise between $n_3=0$ and $n_3=2$, namely $n_3=1$ or $n_1=n_2=n_3=1$. This gives the possibility of an adjoint representation of $SU(3)$ with three different eigenvalues, which has a diagonal $U(1)\times U(1)$ little group; but it is not the stable minimum (against radiative corrections), because if we choose the original possibility ($n_3=0$ or $n_3=2$, i.e., $n_1+n_2=3$), then the extrema of $\mathrm{Tr}(\varphi_r^2)=n_1 a_1^2(1+n_2/n_1)$ yield in both cases (maximum or minimum) the same result, namely

$$\left(\frac{n_1}{n_2}\right)_{\max}: \quad \frac{n_1}{n_2}=\frac{N-1}{1}=2$$

$$\left(\frac{n_1}{n_2}\right)_{\min}: \quad \frac{n_1}{n_2}=\frac{N-[N/2]}{[N/2]}=2, \qquad n_1+n_2=3$$

and this is the stable minimum (against the radiative corrections). The SSB is

$$SU(3)\to SU(2)\times U(1)$$

This is in agreement with our results for the more general potential (1).

APPENDIX

The standard symmetric or antisymmetric matrices introduced in Section 3 are

$$\left[\begin{matrix} \underbrace{\left[\begin{matrix} C & & \\ & \ddots & \\ & & C \end{matrix}\right]}_{n_1}\Bigg\}n_1 & & \\ & 0 & \\ & & \ddots \\ & & & 0 \end{matrix}\right]_{(s)}, \quad \left[\begin{matrix} \underbrace{\left[\begin{matrix} \begin{bmatrix} 0 & C \\ -C & 0 \end{bmatrix} & & \\ & \ddots & \\ & & \begin{bmatrix} 0 & C \\ -C & 0 \end{bmatrix} \end{matrix}\right]}_{n_1}\Bigg\}n_1 & & \\ & 0 & \\ & & \ddots \\ & & & 0 \end{matrix}\right]_{(}$$

the $V(1)$ can be written as

$$V(\varphi_r)=-\alpha n_1 C^2+\beta n_1^2 C^4\pm\gamma n_1 C^4$$

The extremalization relation between C and n is then obtained by

$$\frac{\partial V(\varphi_r)}{\partial C^2}=0, \qquad C^2=\frac{\alpha}{2\beta}\frac{1}{n_1\pm\gamma/\beta}, \qquad C^2\approx\frac{1}{n_1}$$

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